Relating simple harmonic motion and uniform circular motion with Tracker

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In this study, we demonstrate an interesting relationship between the simple harmonic motion and uniform circular motion trough a simple experiment. The experiment requires a low cost, easily-found materials, and free software Tracker. To represent uniform circular motion, we use a tape that is stuck on a fan moving with the constant angular speed. Meanwhile, spring and pendulum motion are used to represent simple harmonic motion. Through video analysis with Tracker , we have shown that the positions (x and y coordinates) of an object undergoes uniform circular motion fit to the sinusoidal function of time, similar to simple harmonic motion. We also analyze the behavior of velocity and acceleration in simple harmonic motion and uniform circular motion. This simple experiment can be used in high school physics courses to lead students in developing a conceptual understanding of uniform circular motion with a less mathematical approach.

Keywords: Simple harmonic motion; uniform circular motion; physics demonstration; Tracker

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1. Introduction

Recently, there is a rapid development of imaging technology. Moreover, technology in mobile phones has been improved significantly. Nowadays, mobile phones provide various features such as cameras, internet, a variety of sensors, spreadsheets, games, and so on. The development of imaging technology and the inclusion of cameras on mobile phones can be used for physics teaching and learning activities. By using imaging technology, Tracker has become a useful tool in physics education. Tracker is a free software developed on the Open Source Physics Java code library which can be used for video analysis and physics modeling. Tracker includes features such as object tracking with position, velocity, and acceleration overlays and graphs, calibration points and line profiles for analysis of spectra and interference patterns [1].

Some studies have explored the potential of Tracker for pedagogical learning tools [2-5]. Poonyawatpornkul and Wattanakasiwich [3] use Tracker to analyze damped harmonic oscillation; Rodrigues and Simeão Carvalho [6] use Tracker to teach optical phenomena; Wee, Tan, Leong, and Tan [5] integrate Tracker in physics class when discussing free-fall motion. Tracker allows students to identify relationships between physical quantities through observation, compare the real world with mathematical modeling, and be trained to construct a model for a particular physical phenomenon [7].

Circular motion and simple harmonic motion are discussed several times in high school physics and introductory college physics courses. Uniform circular motion and simple harmonic motion have close relationships. Simple harmonic motion is a term used when the periodic motion is a sinusoidal function of time. Most students are able to notice simple harmonic motion in the spring-mass system and the simple pendulum. In fact, the $x-$ and $y-$coordinates of particles' positions in uniform circular motion are also a sinusoidal function of time, similar to simple harmonic motion. Sometimes students face difficulty to relate uniform circular motion and simple harmonic motion trough geometric and mathematical approaches.

In this work, we show a simple experiment using Tracker to compare uniform circular motion and simple harmonic motion. Experiments, representing uniform circular motion and simple harmonic motion, have been conducted with simple materials and tools which can be found easily in daily life. The phenomena were recorded with a mobile phone camera. After that, the videos were analyzed through Tracker.

2. Theory

2.1. Simple harmonic motion in vertical spring-mass system

One of the physical systems representing simple harmonic motion is a vertical spring-mass system. A particle with mass m is attached on vertical spring with an original length of l_o such as shown in Fig. 1. At equilibrium point, the spring force is balanced with the gravitational force, or according to Newton's first law, $\boldsymbol{F}_s + \boldsymbol{F}_g = 0$. Thus,

$$-kl + mg = 0,$$

or

$$kl = mg. \tag{1}$$

When the spring is stretched down leaving its equilibrium position, there is acceleration, so that $\sum \boldsymbol{F} = m\boldsymbol{a}$

$$\boldsymbol{F}_s + \boldsymbol{F}_g = m\boldsymbol{a},$$

$$-k(l + y) + mg = m\frac{d^2y}{dt^2}. \tag{2}$$

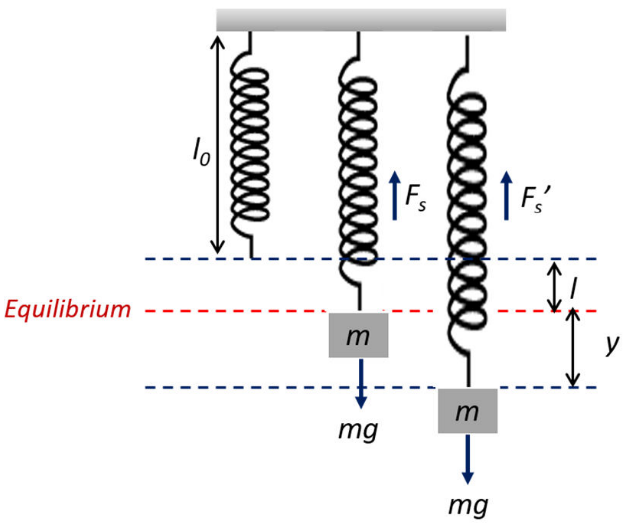

FIGURE 1. Vertical Spring-Mass System.

Since $kl = mg$,

$$-ky = m\frac{d^2y}{dt^2}. \tag{3}$$

Eq. (3) can be written as:

$$\frac{d^2y}{dt^2} = \omega^2 y, \tag{4}$$

where $\omega^2 = k/m$.

By solving the second-order differential equation in Eq. (4); we yield a solution for the particle's position as follows:

$$y(t) = A\cos(\omega t + \phi) \tag{5}$$

where A is the amplitude and ϕ is the initial phase. Both of A and ϕ depend on the initial condition of the particle. As described in Eq. (5), the particle's periodic motion is a sinusoidal function of time, in which the sinusoidal function is a cosine function. It is the main characteristic of simple harmonic motion. Furthermore, we can determine the velocity and acceleration of the particle by taking the first and second derivative of Eq. (5), respectively. Thus,

$$v(t) = \frac{dy(t)}{dt} = -A\omega\sin(\omega t + \phi), \tag{6}$$

$$a(t) = \frac{dv(t)}{dt} = -A\omega^2\cos(\omega t + \phi). \tag{7}$$

2.2. Uniform circular motion

A particle moves within uniform circular motion when it moves in a circular path with a constant angular speed. The angular position of a particle is given as:

$$\theta(t) = \phi + \omega t, \tag{8}$$

where ω is angular speed, whereas ϕ is the initial angular position or initial phase.

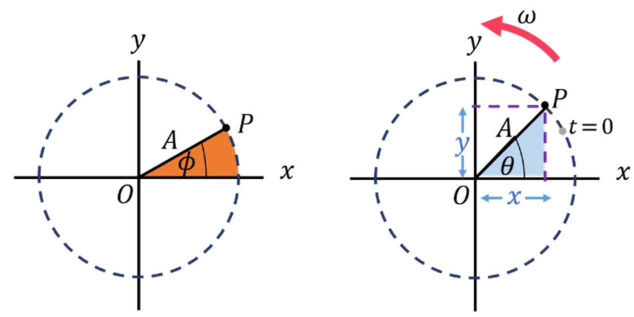

FIGURE 2. Particle P goes through the uniform circular motion with a constant angular speed of ω. The figure on the left is the initial particle's position. The figure on the right side is the particle's position at time t.

Particle P in Fig. 2 is moving in a uniform circular motion. It moves in a circle with a radius of A and constant angular speed ω. The $x-$coordinate of particle P is given by

$$x(t) = A\cos\theta = A\cos(\omega t + \phi). \tag{9}$$

The $x-$coordinate of particle P is a sinusoidal function of time, analogous to the simple harmonic motion. The $x-$component of velocity and acceleration are given in Eqs. (10) and (11), respectively.

$$v_x(t) = \frac{d}{dt}x(t) = -A\omega t\sin(\omega t + \phi) \tag{10}$$

$$a_x(t) = \frac{d}{dt}v_x(t) = -A\omega^2 t\cos(\omega t + \phi) \tag{11}$$

The $y-$coordinates of particle P is also analogous to SHM motion, following the sinusoidal function of time (see Eq. (12)).

$$y(t) = A\sin\theta = A\sin(\omega t + \phi), \tag{12}$$

$$v_x(t) = \frac{d}{dt}y(t) = A\omega\cos(\omega t + \phi), \tag{13}$$

$$a_y(t) = \frac{d}{dt}v_y(t) = -A\omega^2 t\sin(\omega t + \phi). \tag{14}$$

3. Experimental setup

The experimental setup is simple, as shown in Fig. 3. The behavior of uniform circular motion is investigated by recording the motion of tape, which is stuck on an electric fan (see Fig. 3a). The recorded video is then analyzed with Tracker. Through Tracker, we get data about position, velocity, and acceleration as a function of time. Meanwhile, the behavior of simple harmonic motion is represented by the motion of a slotted weight which is attached to a vertical spring (see Fig. 3b).

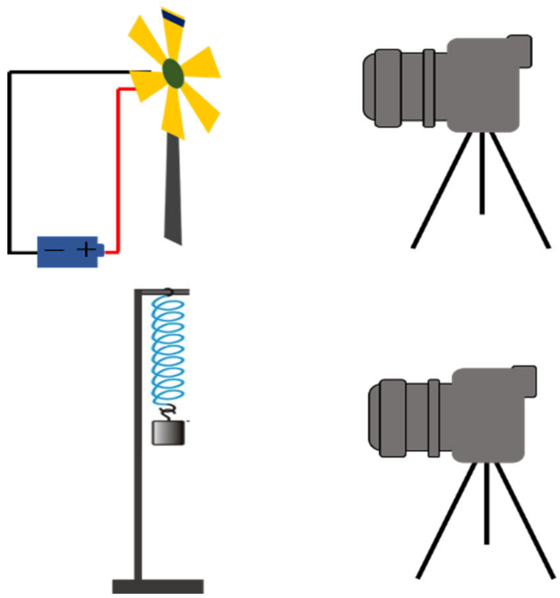

FIGURE 3. Experimental set-up of (a) uniform circular motion using an electrical fan, (b) simple harmonic motion using spring and slotted weight.

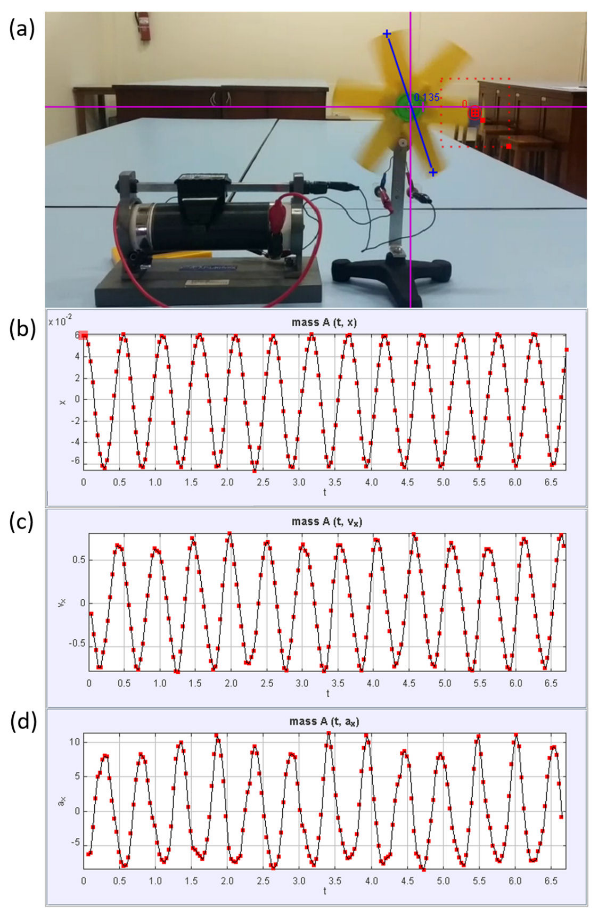

FIGURE 4. (a) Uniform circular motion experiment using an electric fan, (b) $x-$position versus time graph, (c) $x-$component linear velocity versus time graph, (d) $x-$component acceleration versus time graph in a uniform circular motion. All quantities are presented in SI units.

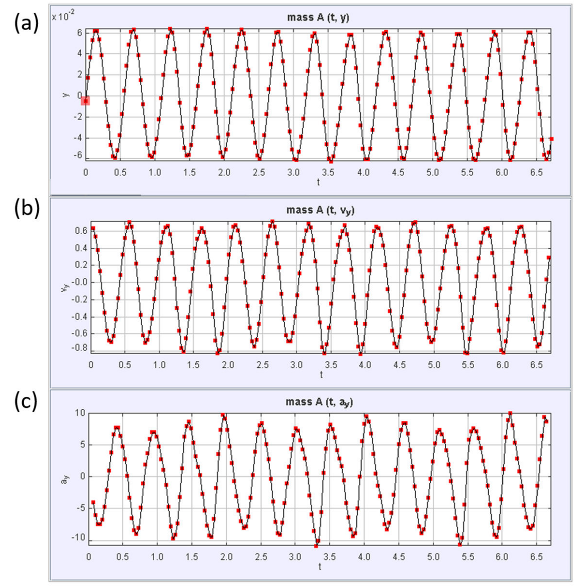

FIGURE 5. (a) $y-$position versus time graph, (b) $y-$component linear velocity versus time graph, (c) $y-$component acceleration versus time graph in a uniform circular motion. All quantities are presented in SI units.

4. Result and discussion

The position of an object attached to a fan which moves with constant angular speed has been analyzed through Tracker. As time goes on, the $x-$coordinate of the object goes down and up following cosine function (see Fig. 4b) as predicted in theory. Although it is not perfectly smooth, the $x-$component of linear velocity tends to follow sine function with an initial phase (ϕ) of π radians. Meanwhile, the $x-$component of acceleration tends to follow negative cosine function, consistent with Eq. (11).

As shown in Fig. 5, the y-coordinate of the object on uniform circular motion smoothly fits with the sine function in Eq. (12). On the other hand, the $y-$component of velocity and acceleration graph are not as smooth as the position graph. However, the $y-$component of velocity and acceleration still tend to follow cosine and negative sine function, respectively. These results demonstrate that uniform circular motion has symmetry with simple harmonic motion.

In addition, we performed an experiment with a well-known simple harmonic motion system, *i.e.* mass-spring system and simple pendulum. Figure 6b-d) shows the position, velocity, and acceleration of a slotted weight attached on a vertical spring as functions of time. The spring was initially stretched down. As predicted, the position of slotted weight fits sine function with $\phi = -\pi$ radians. The velocity fits cosine function with $\phi = -\pi/2$, whereas the acceleration fits cosine function or sine function with $\phi = \pi/2$.

The analysis of simple pendulum motion is presented in Fig. 7. Consistent with the mass-spring system, the periodic motion of the pendulum is a sinusoidal function of time. The

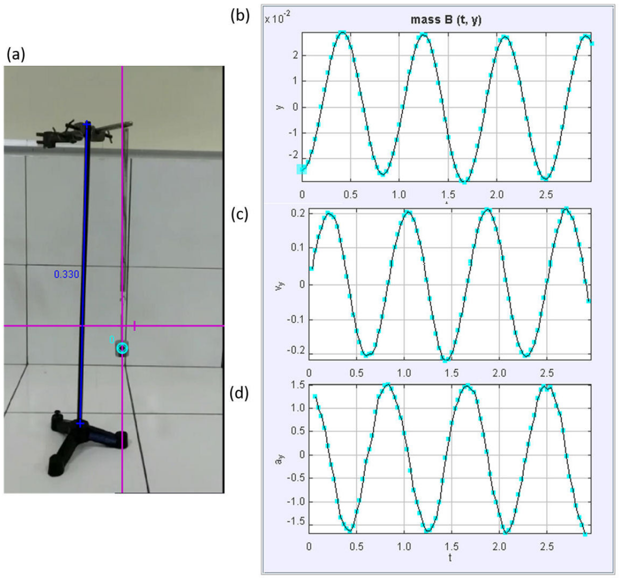

FIGURE 6. (a) simple harmonic motion experiment using spring and slotted weight, (b) $y-$position, (c) $y-$component velocity, (d) $y-$component acceleration of mass as functions of time in the simple harmonic motion of a spring-mass system. All quantities are presented in SI units.

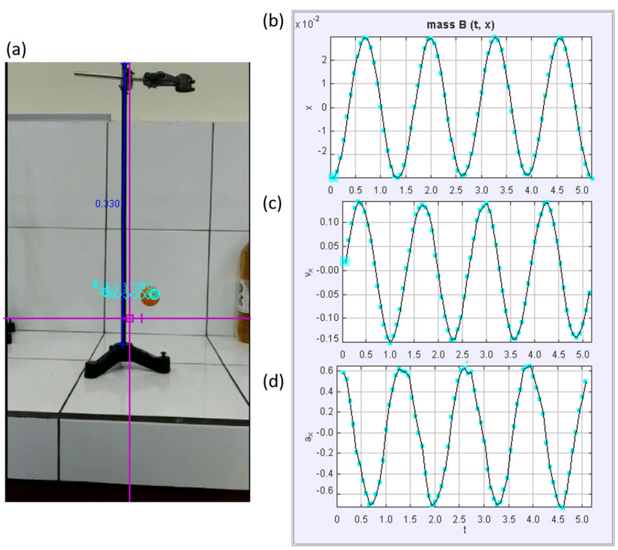

FIGURE 7. (a) simple pendulum experiment, (b) $x-$position, (c) $x-$component velocity, (d) $x-$component acceleration of mass as functions of time in a simple harmonic motion of simple pendulum system. All quantities are presented in SI units.

ball was swung to the left initially. The position of the ball perfectly follows the sine function with $\phi = \pi$ radians. The velocity fits cosine function with $\phi = -\pi/2$. Meanwhile, the acceleration graph fits the sine function with $\phi = \pi/2$.

We have shown through analysis with Tracker that uniform circular motion has symmetry with simple harmonic motion. The linear position, velocity, and acceleration in uniform circular motion fit the sinusoidal function of time, which is the same in simple harmonic motion.

5. Strategic implementation in class

A simple experiment with Tracker could potentially be included in physics classrooms to enhance students' conceptual understanding in a uniform circular motion and simple harmonic motion. There are some possible ways to implement this simple experiment in the classroom. For example, the teacher can bring a recorded video about uniform circular motion and simple harmonic motion in class. At the beginning of the class, the videos are distributed to students. With teacher guidance, students are asked to analyze the videos with Tracker. In the scenario, students, individually or in a group, can record the uniform circular motion and simple harmonic motion by themselves with the teacher's help. This scenario will engage students more in the experiments and hopefully, students become more motivated in learning physics.

The teacher can make this activity as project, laboratory activity, homework or class demonstration. After the activity, the teacher can discuss the relation between simple harmonic motion and uniform circular motion in class through a mathematical approach. The discussion will enrich the students' conceptual knowledge about simple harmonic motion and circular motion.

6. Conclusion

In summary, we have demonstrated the relationship between uniform circular motion and simple harmonic motion by a simple experiment using Tracker with non-mathematical approach. The equation of motion of an object which moves in a uniform circular motion is analog to an object which moves in simple harmonic motion such as in spring-mass or pendulum system. The $x-$ and $y-$coordinates of an object undergoes uniform circular motion, fit to the sinusoidal function of time. The velocity and acceleration also fit the sinusoidal function of time. In other words, we can say that simple harmonic motion can be considered as a projection of uniform circular motion. This demonstration can be embedded in high school physics classes to enhance students' conceptual understanding of simple harmonic motion and uniform circular motion.

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